

Nucleon TMDs from Lattice QCD

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In collaboration with

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Abstract

I present lattice QCD calculations of transverse momentum dependent parton distribution functions (TMDs) of protons using operators with staple-shaped Wilson lines. For naively time-reversal odd observables, we study the generalized Sivers and Boer-Mulders transverse momentum shift applicable to SIDIS and DY experiments, and for T-even observables we calculate the transversity related to the tensor charge and the generalized worm-gear shift. Results obtained on two different $nf = 2 + 1$ flavor ensembles, domain-wall fermion (DWF) with lattice spacing 0.084 fm and pion mass of 297 MeV, and clover fermion with lattice spacing 0.114 fm and pion mass of 317 MeV will be compared and current level of control over systematics highlighted. Agreement between the two formulations indicates that the assumption that soft and renormalization factors cancel in ratios of functions is reasonable.

TMDs

- Transverse Momentum Dependent Parton Distribution Functions
- Probability to find a quark carrying longitudinal momentum fraction x and transverse momentum \mathbf{k}_\perp in a fast moving nucleon
- Eg., Unpolarized function $f_1^q(x, k_\perp)$; $\int d^2\mathbf{k}_\perp f_1^q(x, k_\perp) = f_1^q(x)$

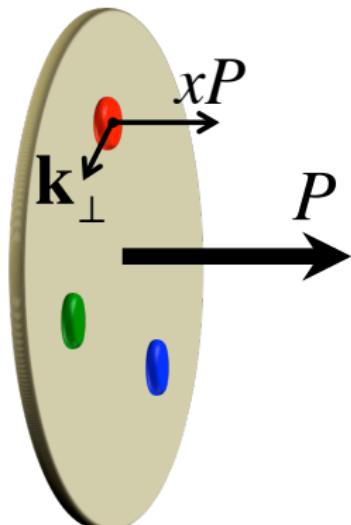
| Nucleon | | |
|-------------|-----------------------------|-----------------------------|
| Quark | Unpolarized | L-pol |
| Unpolarized | f_1 | |
| L-pol | | g_{1L} Helicity |
| T-pol | h_1^\perp Boer-Mulders | h_{1L}^\perp Worm-gear |

Sivers

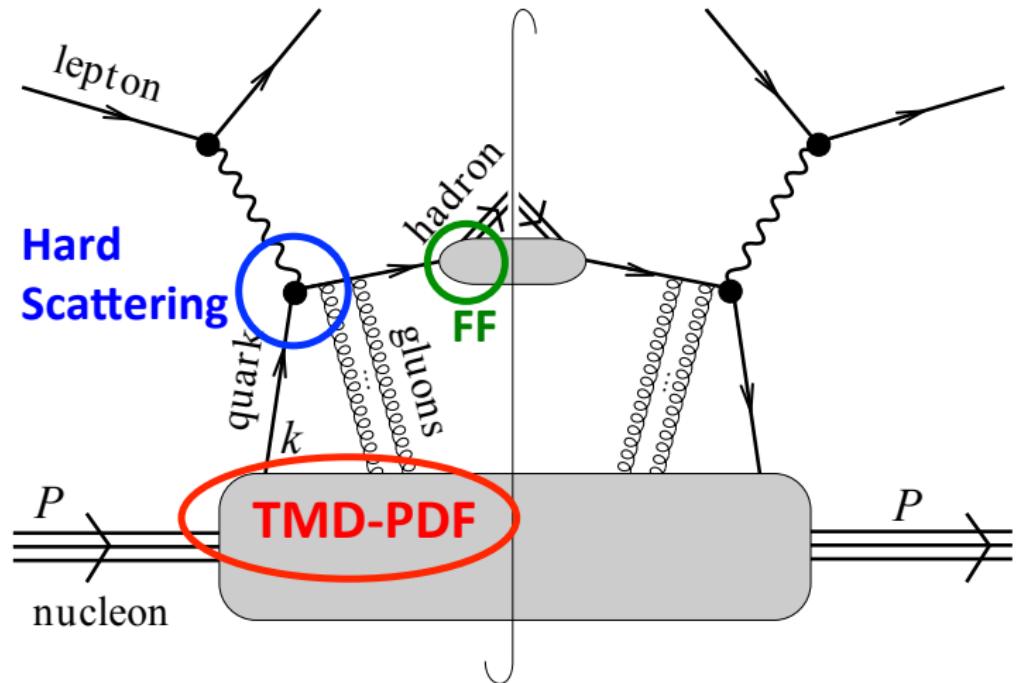
Worm-gear

Transversity

Pretzelosity



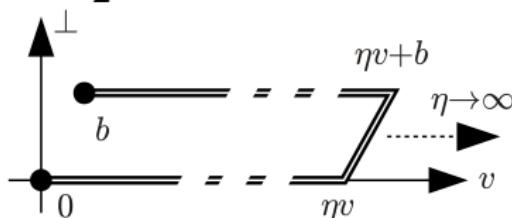
Semi-inclusive Deep Inelastic Scattering (SIDIS)



Cross Section = [TMD-PDF] \otimes [Hard Scattering] \otimes [FF]

Lattice Calculation of TMDs

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$



- Parameterize correlation functions on the lattice in terms of Lorentz-invariant amplitudes
- Staple geometry T-odd for SIDIS / DY processes
- Take v off the light-cone into space-like region
- Cast Euclidean results \Rightarrow TMDs in light-cone coordinate
- Perturbative evolution governs the approach to the light-cone
[Aybat, Collins, Qiu and Rogers, 2012]
- Take the light-cone limit in Minkowski space: $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|} \rightarrow \infty$

TMDs from Matrix Elements

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

$$\Phi^{[\Gamma]}(x, k_T, \dots) = \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} e^{ix(b \cdot P) - ib_T \cdot k_T} \frac{\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, \dots)}{\tilde{\mathcal{S}}(b^2; \dots)}$$

- $\tilde{\mathcal{S}}$: “Soft factor” needed to cancel divergence from \mathcal{U}
- Parameterize in terms of Lorentz-invariant amplitudes to extract TMDs from the Lattice calculation
- $\Phi^{[\Gamma]}(x, k_T, \dots)$ can be decomposed in terms of TMDs as

$$\Phi^{[\gamma^+])(x, k_T, S, \dots) = f_1(x, k_T) - \frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp(x, k_T)$$

Lattice Parameters

| Fermion Lattices from | DWF on AsqTad ¹ | DWF ² | Clover |
|-----------------------|----------------------------|------------------|----------|
| a (fm) | MILC | RBC/UKQCD | JLab/W&M |
| m_π (MeV) | 0.12 | 0.084 | 0.114 |
| Statistics | 518, 369 | 297 | 317 |
| | 3888, 5264 | 4264 | 23208 |

- Understanding systematics regarding
 - Finite lattice spacing
 - Pion mass
 - Chiral/Non-chiral fermion discretization
 - Non-local operator renormalization

¹Musch, *et al.*, PRD 2012

²Engelhardt, Lattice 2014

TMDs Calculated

- **T-Even TMDs**

- Transversity h_1^\perp
- Worm-gear g_{1T}

- **T-Odd TMDs**

- Sivers f_{1T}^\perp
- Boer-Mulders h_1^\perp

| | | Nucleon | | |
|-------|-----------------------------|-----------------------------|-----------------------------|--------------------------------|
| | | Unpolarized | L-pol | T-pol |
| Quark | Unpolarized | f_1 | | f_{1T}^\perp Sivers |
| | L-pol | | g_{1L} | g_{1T} |
| | T-pol | | Helicity | Worm-gear |
| T-pol | h_1^\perp Boer-Mulders | h_{1L}^\perp Worm-gear | h_1^\perp Transversity | h_{1T}^\perp Pretzelosity |

Generalized Shifts

- **Generalized Shifts**

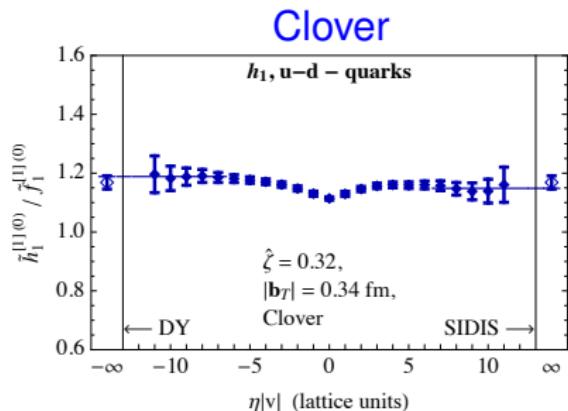
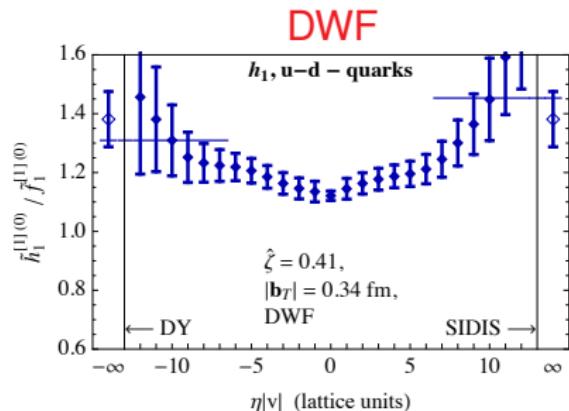
$$\text{Sivers : } \langle \mathbf{k}_y \rangle_{TU}(\mathbf{b}_T^2, \dots) = m_N \frac{\tilde{f}_{1T}^{\perp1}(\mathbf{b}_T^2, \dots)}{\tilde{f}_1^{[1](0)}(\mathbf{b}_T^2, \dots)}$$

$$\text{Boer-Mulders : } \langle \mathbf{k}_y \rangle_{UT}(\mathbf{b}_T^2; \dots) = m_N \frac{\tilde{h}_1^{\perp1}(\mathbf{b}_T^2; \dots)}{\tilde{f}_1^{[1](0)}(\mathbf{b}_T^2; \dots)}$$

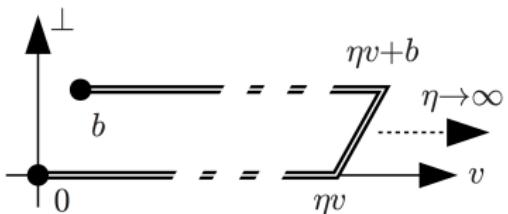
$$-\tilde{f}^{[1]} = \int_{-1}^1 dx \tilde{f}(x); \quad \tilde{f}^{(1)}(x, b_T^2) = n! \left(-\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2)$$

– Soft factors and multiplicative renormalization factors cancel

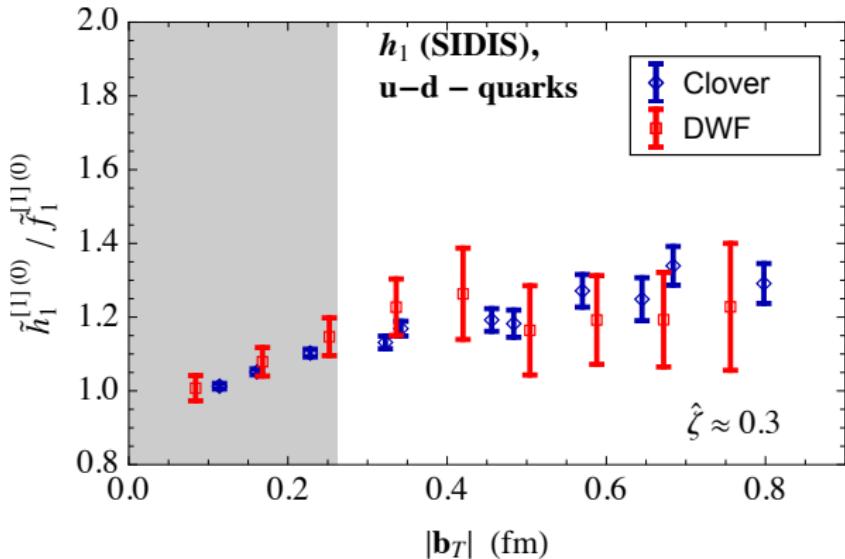
Transversity: Dependence on Staple Extent



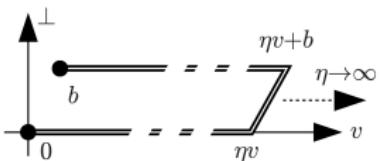
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-even, SIDIS}} = f^{\text{T-even, DY}}$



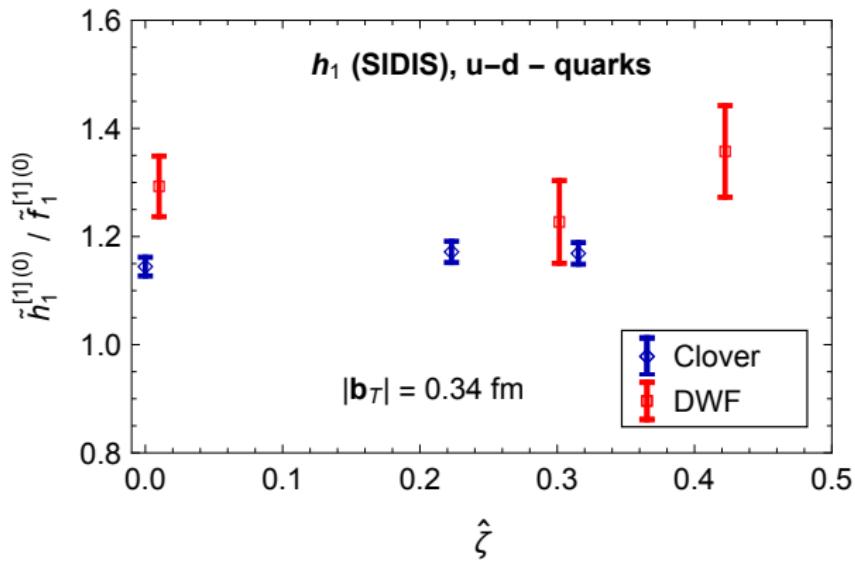
Transversity: Dependence on $|b_T|$



- b_T : Fourier conjugate of k_T

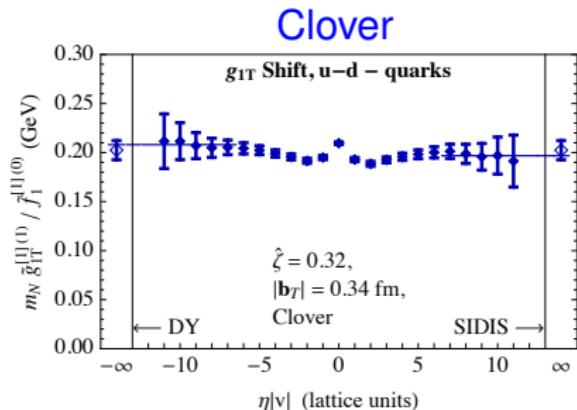
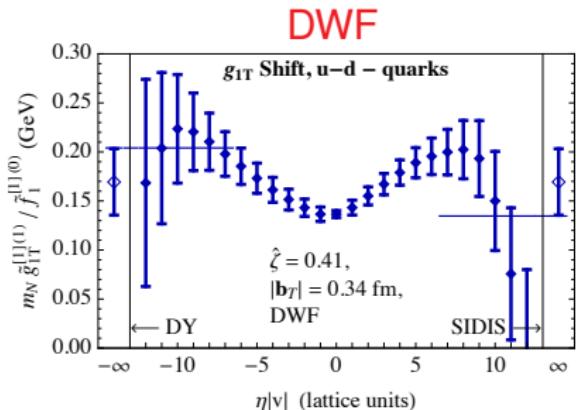


Transversity: Dependence on $\hat{\zeta}$

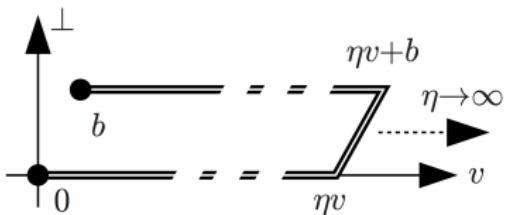


- Collins-Soper parameter $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|}$
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$

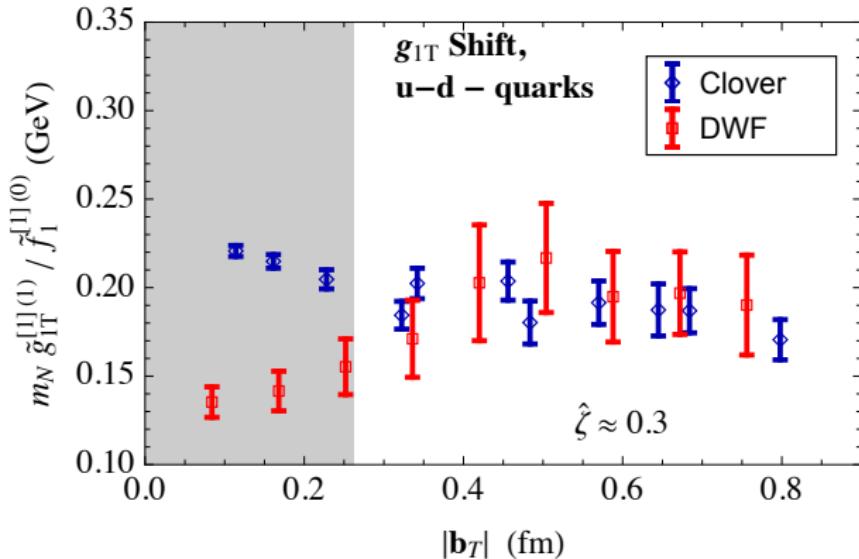
Worm-gear Shift: Dependence on Staple Extent



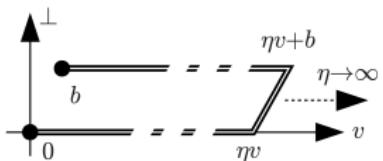
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-even, SIDIS}} = f^{\text{T-even, DY}}$



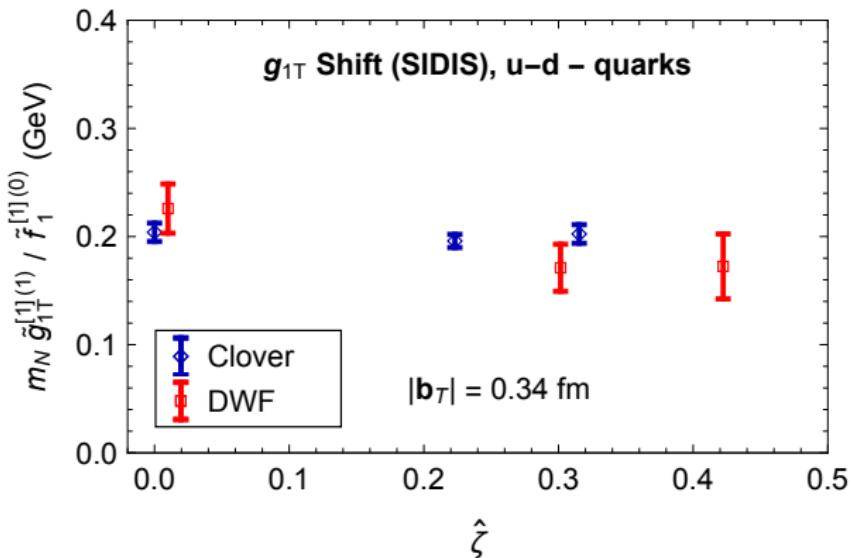
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- b_T : Fourier conjugate of k_T



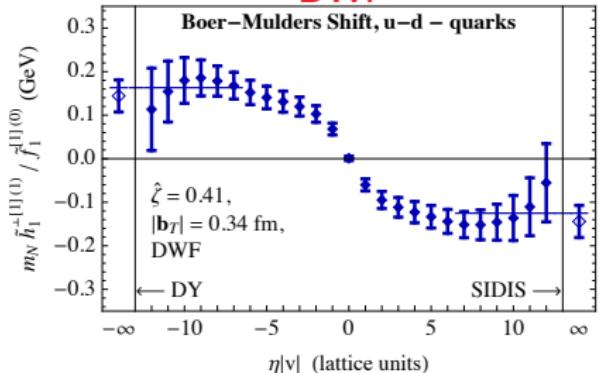
Worm-gear Shift: Dependence on $\hat{\zeta}$



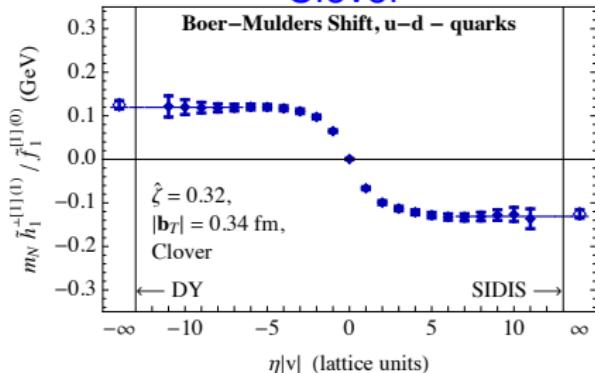
- Collins-Soper parameter $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|}$
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$

Boer-Mulders Shift: Dependence on Staple Extent

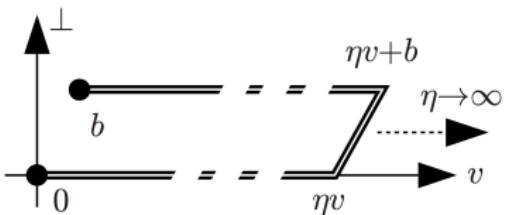
DWF



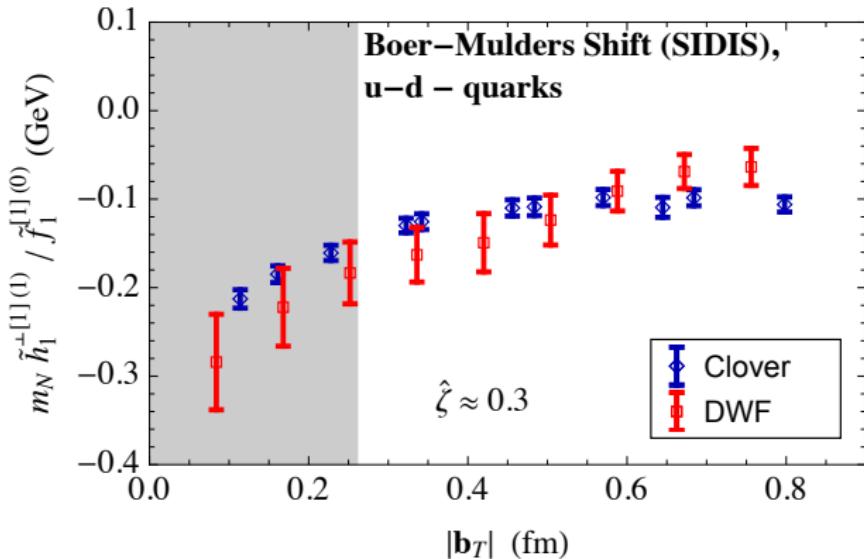
Clover



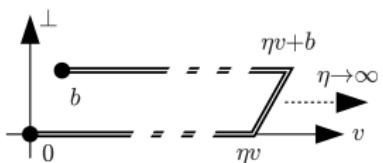
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$



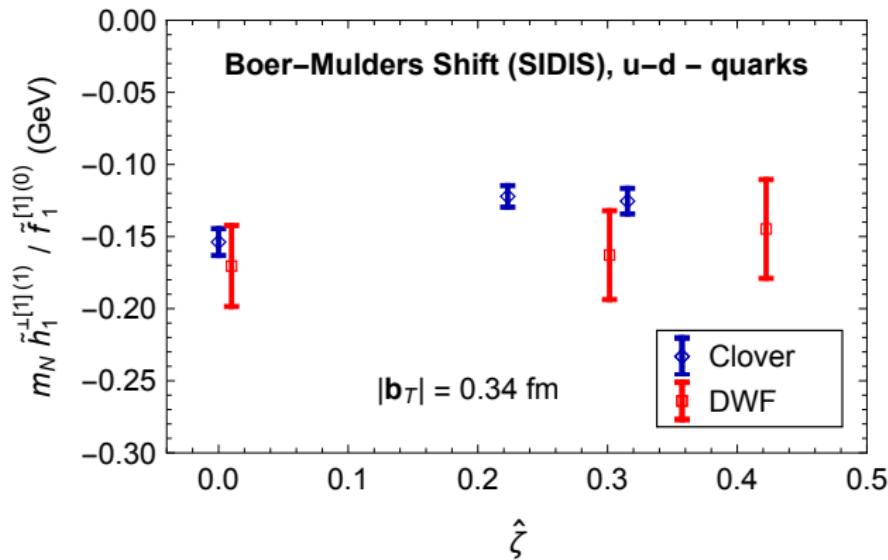
Boer-Mulders Shift: Dependence on $|b_T|$



- b_T : Fourier conjugate of k_T

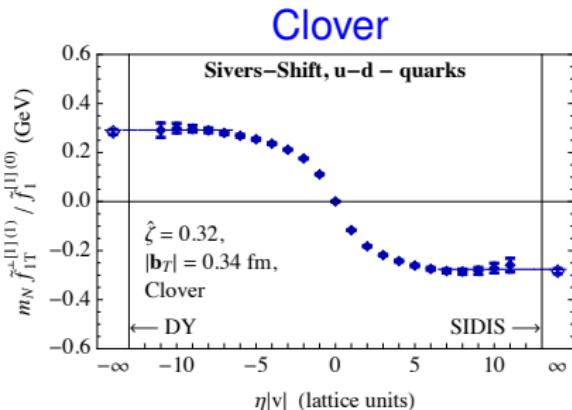
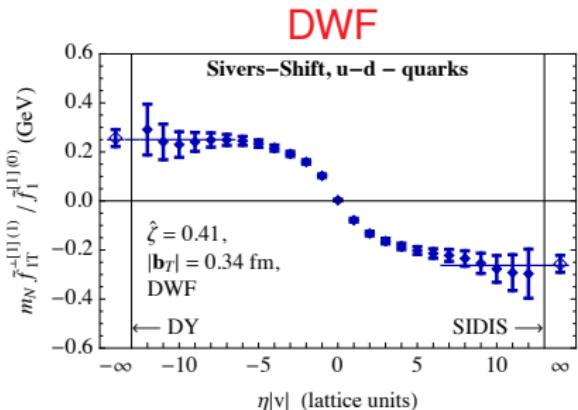


Boer-Mulders Shift: Dependence on $\hat{\zeta}$

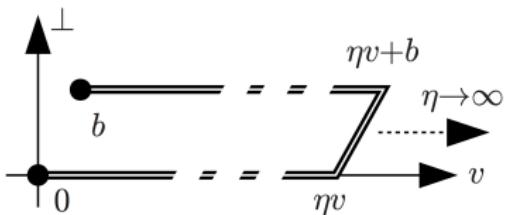


- Collins-Soper parameter $\hat{\zeta} \equiv \frac{\mathbf{v} \cdot \mathbf{P}}{|\mathbf{v}| |\mathbf{P}|}$
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$

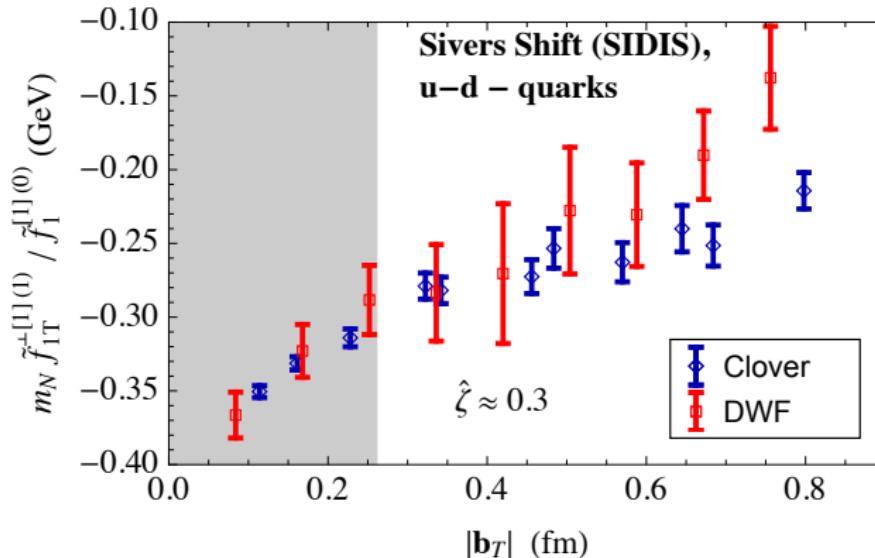
Sivers Shift: Dependence on Staple Extent



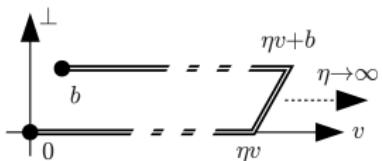
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$



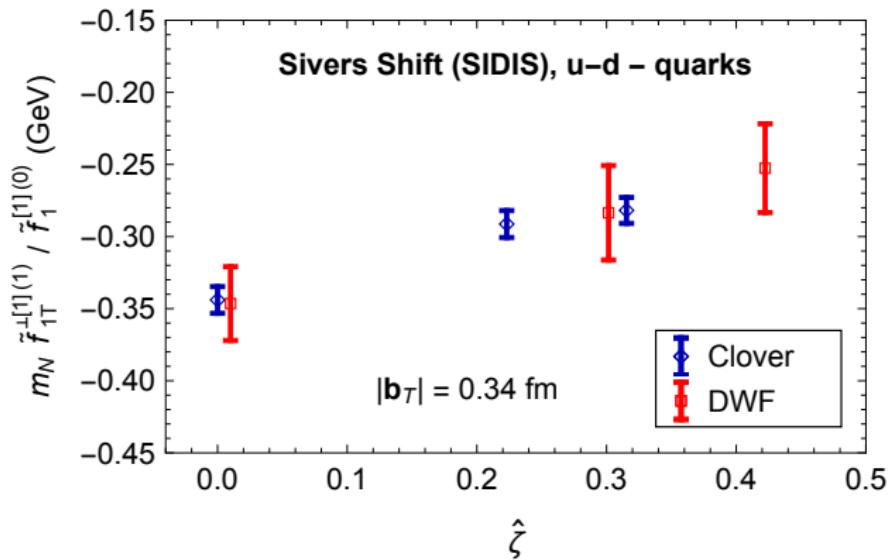
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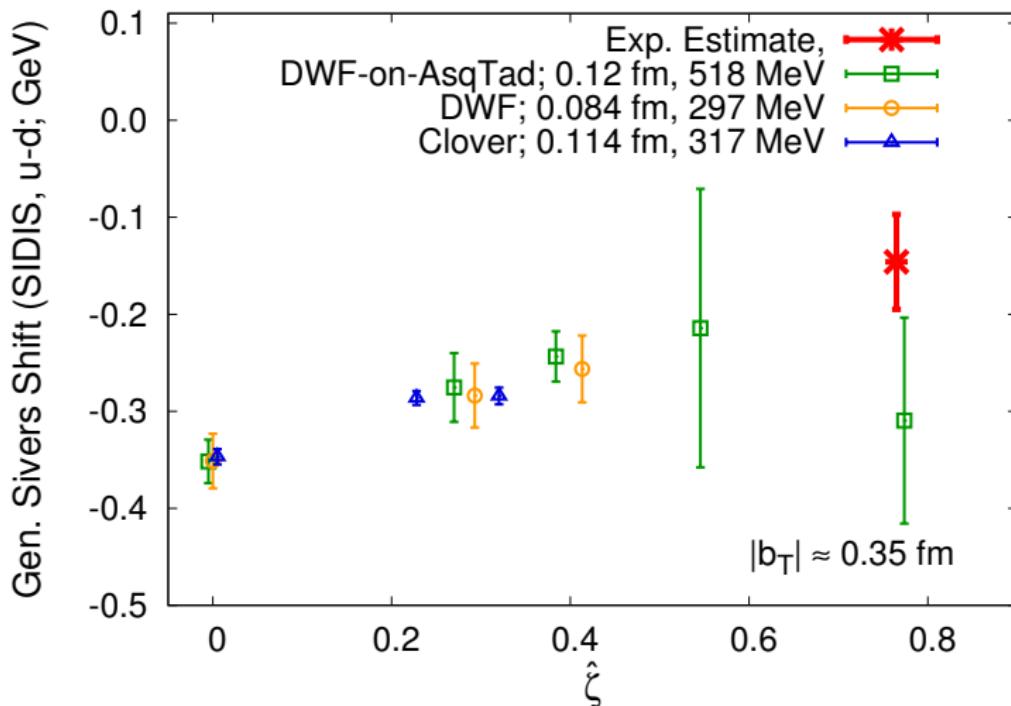


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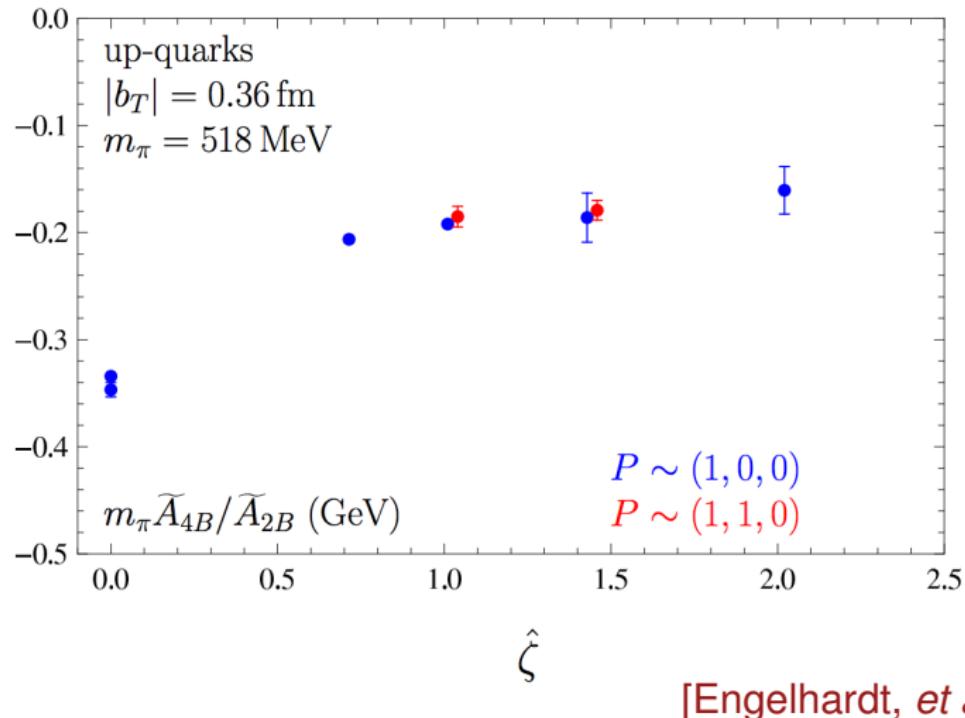
- Collins-Soper parameter $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|}$
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$

Comparison with Experiment

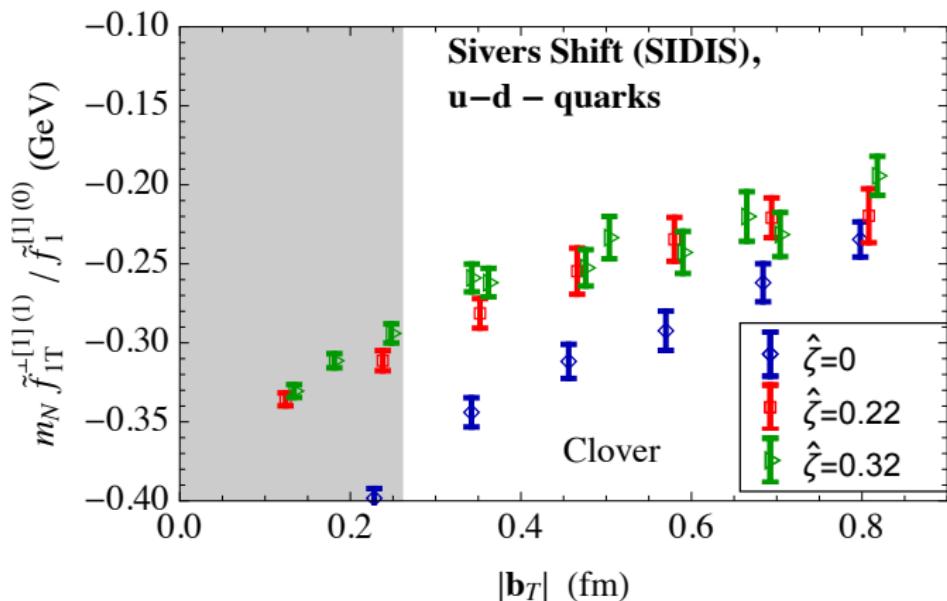


- Extraction from experiment based on [Echevarria, *et al.*, 2014]

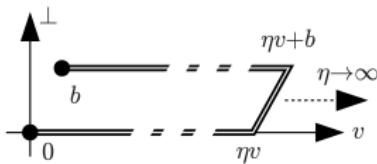
Pion: Boer-Mulders Shift for Large- $\hat{\zeta}$



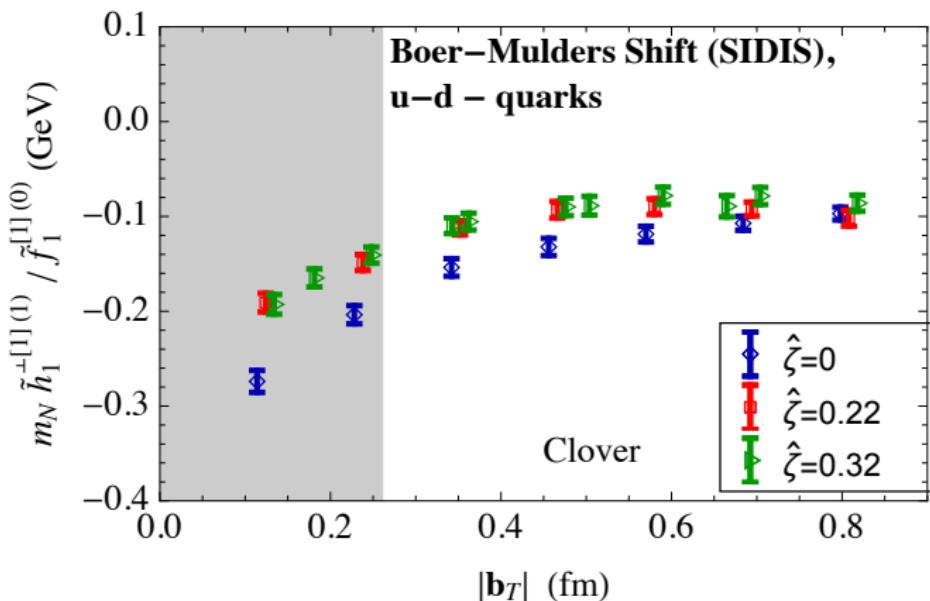
Nucleon: Different $\hat{\zeta}$ data converge when b_T is large



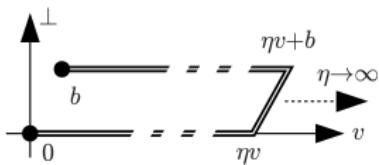
- b_T : Fourier conjugate of k_T
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$



Nucleon: Different $\hat{\zeta}$ data converge when b_T is large



- b_T : Fourier conjugate of k_T
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$



Summary: TMDs using lattice QCD

- Investigated lattice discretization effects at $M_\pi \approx 300$ MeV
- No significant difference in TMDs between $a = 0.084$ fm DWF and $a = 0.114$ fm Clover lattices at $M_\pi \approx 300$ MeV.
→ Indicating cancellation of soft and renormalization factors is a reasonable assumption for $b_T > 0.25$ fm
- Comparison of Sivers shift with phenomenology:
Lattice data tend towards the Experiment value at $\hat{\zeta} = 0.77$.
Need more studies at higher $\hat{\zeta}$ and smaller M_π .
- Need to develop methods to improve signal at large momenta to access large $\hat{\zeta}$